Math 2010 Prof. Y. Kwong Assignment IV

1/. Given $f(x,y) = \begin{cases} x(y+1)\ln(x^2+y^2) & \text{if } (x,y) \neq (0,0), \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$

Is f continuous at $(0,0)$? If not, is the discontinuity removable?

2/. $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{at } (x,y) = (0,0) \end{cases}$

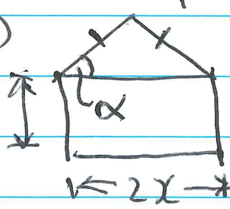
Compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ (Note that you couldn't differentiate directly).

3/. Given $u = u(x,y)$, turn (x,y) into polar co-ordinates (r,θ) i.e. $x = r\cos\theta$, $y = r\sin\theta$, then $u = u(r,\theta)$. Prove the identity:

$$\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

4/. Given a plane $Ax + By + Cz + D = 0$ where $D \neq 0$. Find the point (x,y,z) on the plane that is nearest to the origin $(0,0,0)$.

5/. A pentagon is composed of a rectangle surmounted by an isos. Δ (see diagram)



where α is the base angle of the isos. Δ. If the pentagon has a fixed perimeter P, find the maximum possible area of the pentagon.

6/. Prove that every plane tangent to the upper cone $z = \sqrt{x^2+y^2}$ passes through the origin $(0,0,0)$.

7/. Find all the planes that are tangent to both of the sphere $x^2+y^2+z^2 = 8$ and the vertical cylinder $xy = 4$, $-\infty < z < \infty$.

8/. Find all functions $f(x,y,z)$ such that $\frac{\partial f}{\partial x} = 2xy + z^2$, $\frac{\partial f}{\partial y} = 2yz + x^2$, and $\frac{\partial f}{\partial z} = 2xz + y^2$. Before you start to look for $f(x,y,z)$, conduct a preliminary test to make sure this is possible.